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Free convection in inclined air layers heated from above

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Abstract—An experimental investigation of steady-state natural convection heat transfer was carried out in finite rectangular air layers heated from above. Two different aspect ratios, namely $A = 20$ and 80, and perfectly conducting boundary conditions on the end walls were used. The angle of inclination was varied from $\phi = 0$ (heated from below) to $\phi = 180^\circ$ (heated from above). A total of 226 test points were taken for heat transfer measurements in air layers heated from above at four different orientations in the range $120 \leqslant \phi \leqslant 180^\circ$ for Rayleigh numbers between 10^2 and 2×10^6. Additional test points have been carried out to show the effect of the angle of tilt in the range $0 \leqslant \phi \leqslant 180^\circ$ on the average Nusselt number for fixed values of the Rayleigh number. Local measurements of the Nusselt number over discrete portions of the air layer are reported to show the Nusselt number distribution over different flow regimes. Copyright © 1996 Elsevier Science Ltd.

1. INTRODUCTION

The problem of steady-state two-dimensional natural convection in an enclosed tilted rectangular cavity heated from below has been the subject of several numerical, analytical and experimental studies. A comprehensive review is given by Catton [1]. In addition, the problem of increasing heat transfer by free convection in a liquid or gas-filled cavity by heating from above is of great interest. However, this case of tilted layers heated from above has received little attention and a serious lack of experimental data in this area is noticed. Figure 1 shows a sketch of a tilted rectangular air layer. It consists of two parallel flat plates with one surface heated at a constant temperature of T_h and the other one cooled at a constant temperature of T_c. The cavity is closed at the ends by surrounding a solid strip of thickness B and thermal conductivity K_w. The cavity aspect ratio is defined as the ratio of the height H to the plate spacing L ($A = H/L$). The thermal boundary conditions on the end walls can be defined through the ratio of the end walls and fluid thermal conductivities K_w/K and the ratio L/B. Koutsoheras and Charters [2] studied the effect of the end walls thermal boundary conditions on the natural convection across air cavities. Two extremes are known as limits for the boundary conditions. These are the perfectly insulating or zero heat flux (ZHF) and perfectly conducting or linear temperature profile (LTP) end walls. The two-dimensional numerical study of Koutsoheras and Charters [2] covered the range $1 \leqslant L/B \leqslant 10$ and $0 \leqslant K_w/K \leqslant \infty$ for small aspect ratios $1/3 \leqslant A \leqslant 3$. They reported heat transfer results only for $A = 1$ and $0 \leqslant \phi \leqslant 90^\circ$ (heated from below).

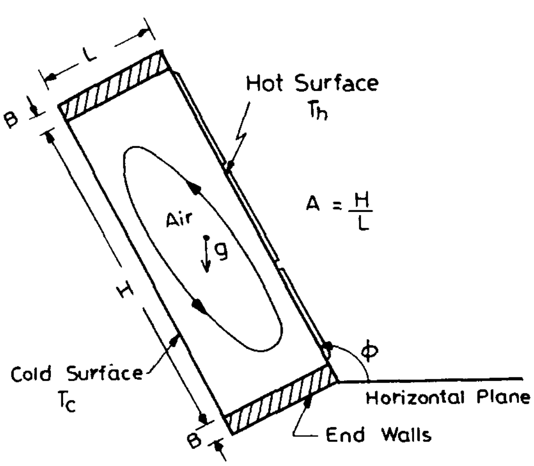

Fig. 1. A sketch of the problem.

Catton *et al.* [3] used the Galerkin method to solve the two-dimensional steady flow of infinite Prandtl number in tilted rectangular cavities. They reported results in the range $0.2 \leqslant A \leqslant 20$ and $60 \leqslant \phi \leqslant 165^\circ$ for both adiabatic and perfectly conducting boundary conditions on the border strips. They found that thermal instabilities associated with regions of adverse temperature gradient (due to the presence of perfectly conducting end walls) should be possible, even in the case of top heating which is known as a convectively stable regime. These effects would be more pronounced for the case of finite Prandtl number. An experimental study was performed by Arnold *et al.* [4] using high Prandtl number fluids (silicon oils and water) in layers of aspect ratios $A = 1$, 3, 6 and 12. The angle of tilt was varied between $\phi = 0$ (heated from below) to $\phi = 180^\circ$ (heated from above) and measurements were taken for Rayleigh numbers between 10^3 and 10^6. They derived the following sca-

NOMENCLATURE

A	aspect ratio ($A = H/L$)	T_c	temperature of cold plate
B	thickness of end walls	T_h	temperature of hot plate.
g	gravitational acceleration		
h	average heat transfer coefficient between isothermal plates		
H	plate height		Greek symbols
K	thermal conductivity of air	α	thermal diffusivity of air
K_w	thermal conductivity of end walls	β	volumetric thermal expansion coefficient of air
L	plate spacing	ΔT	plate temperature difference, $\Delta T = T_h - T_c$
Nu	average Nusselt number, $Nu = hL/K$	ϕ	angle of tilt from horizontal
Pr	Prandtl number, $Pr = \nu/\alpha$	ν	kinematic viscosity of air.
Ra	Rayleigh number, $Ra = L^3\beta g\Delta T/\nu\alpha$		

ling law for angles of inclination from $\phi = 90°$ (vertical layers) to $\phi = 180°$ (heated from above)

$$Nu(\phi) = 1 + [Nu(\phi = 90°) - 1] \cdot \sin\phi. \quad (1)$$

An experimental and numerical study was given by Ozoe *et al.* [5] for high Prandtl number ($Pr = 10$) in an inclined square cavity. The case for adiabatic boundary conditions was tested in a limited range of Rayleigh numbers between 2000 and 8000 for layers heated from below $0 \leqslant \phi \leqslant 90°$. A local maximum in the average Nusselt number was found to occur at about 50° of inclination. This study was extended by Ozoe *et al.* [6] for higher values of the aspect ratio $A = 1$–4, and the angle of inclination was varied between 0 and 180°. Their experimental results using silicon oils ($Pr = 4045$) were in good agreement with the numerical predictions.

The only previous comprehensive study for inclined air layers heated from above was given in the Housing Research Paper no. 32 [7]. This work was done at the National Bureau of Standards, U.S.A. using air layers of aspect ratio in the range $18 \leqslant A \leqslant 96$. The paper presented the measurements of heat transfer for air spaces of various emissivities and thicknesses. A total of 146 test points were taken for air spaces at five different orientations in the range $0 \leqslant \phi \leqslant 180°$ using an apparatus of the guarded hot box type. The experimental results were given as plots of the value hL against ΔTL^3 for fixed values of the angle of inclination. Tabor [8] reduced the results given in ref. [7] to the usual nondimensional groups of Nusselt against Grashof numbers. Hamady *et al.* [9] studied experimentally and numerically the effect of inclination on local and average Nusselt numbers in a square cavity filled with air in the Rayleigh number range between 10^4 and 10^6. For $\phi = 180°$ (heated from above) and $Ra = 1.1 \times 10^5$, their results show clear evidence of the contribution of convection to the heat transferred at the end walls, which results in an average Nusselt number greater than one. Recently, Kuyper *et al.* [10] presented a two-dimensional numerical study of the natural convection of air in an inclined cavity for both laminar and turbulent flows. However, their study was limited to a square cavity with the angle of inclination varied from zero to 180° and Rayleigh numbers between 10^4 and 10^{11}. The purpose of the present work is to provide a definitive study and heat transfer results for natural convection in tilted air layers with large aspect ratios and heated from above.

2. TEST APPARATUS

The present investigation was carried out on the same test apparatus built by the author and fully explained in a previous study [11]. As shown in Fig. 2, it consists of two parallel flat copper plates each measured 635 by 635 by 12.7 mm thickness in overall dimensions. In the present experiments, the heat flux across the air cavity was measured over the full length H of the hot plate. This was done by inserting three electrically heated heater plates into three recesses along the vertical centerline of the hot plate. Each heater plate measured 200 by 200 by 3.1 mm thickness in overall dimensions. A heat flux meter was inserted below each heater plate. Thus, an average value of the heat flux was measured for each third of the plate height. To maintain isothermal surfaces on the copper plates, each of the hot and cold plates was connected to a constant temperature water bath, with its thermostat set at fixed, but different, temperature. The hot or cold water was circulated through tubes soldered to the back of each copper plate. A thermopile consisting of six copper–constantan thermocouples embedded in each plate was used to measure the temperature difference ΔT between the two copper plates. Perfectly conducting boundary conditions on the end walls were established by surrounding the air layer at the four sides by a copper sheet of 0.50 mm thickness.

The Rayleigh number was varied by varying the air pressure inside the cavity, while the spacing L and the temperature difference ΔT were kept unchanged. This method was used to separate the effect of the Rayleigh number on Nusselt number from any other effects due to the variation in aspect ratio or thermal properties

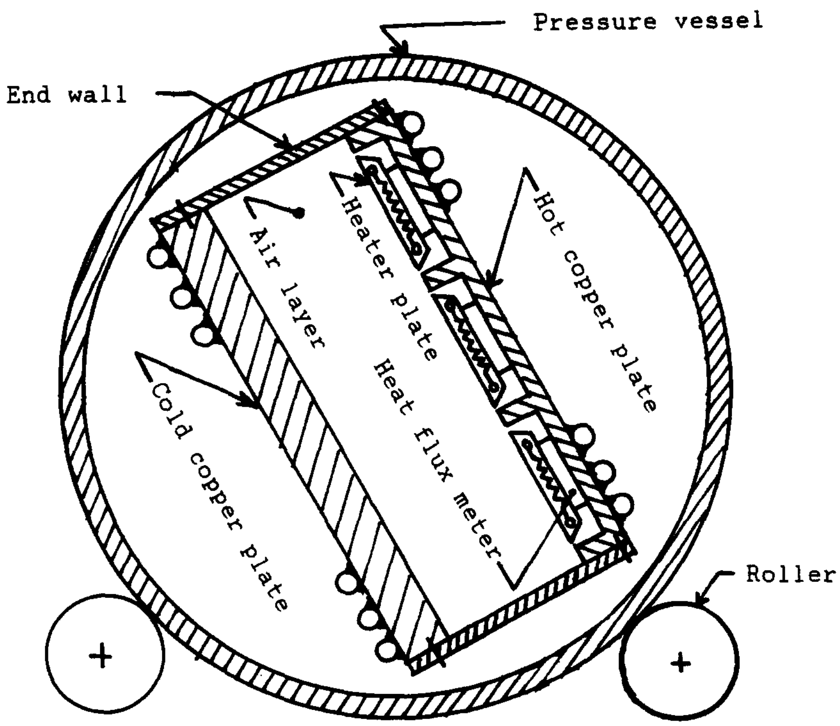

Fig. 2. A sketch of test apparatus.

of air. Therefore, the two plates were enclosed in a pressure vessel, where the pressure can be varied from 100 Pa to 0.7 MPa. The pressure vessel can be rotated around a horizontal axis to give any required angle of tilt.

3. RESULTS AND DISCUSSION

In the present study, heat transfer measurements were taken for inclined rectangular air layers heated from above with two different fixed values of the aspect ratio $A = 20$ and 80. Perfectly conducting boundary conditions were used on the end walls. Figure 3 shows the data for both aspect ratios for air layers tilted at 120 and 135° from the horizontal. For $\phi = 120°$, it is clear that conduction regime prevailed up to $Ra = 3000$ and then the Nusselt number started to deviate from the value of pure conduction ($Nu = 1$) until the data finally took a one third slope at $Ra \geqslant 3 \times 10^4$, indicating a change to the turbulent flow regime. Increasing the aspect ratio from 20 to 80 resulted in a reduction in the Nusselt number. For $\phi = 135°$, the same general behaviour was obtained with departure from pure conduction at a slightly higher Rayleigh number for the higher aspect ratio. The experimental data from ref. [7] are plotted on the same figure for $\phi = 135°$. It should be noted that the Rayleigh number for the data from ref. [7] was increased by increasing the plate spacing L; i.e. decreasing the aspect ratio. Thus, no aspect ratio effect was detected in their data. The dashed line representing the data from ref. [7] for $18 \leqslant A \leqslant 96$ follows the present data for high aspect ratio ($A = 80$) at low Ra, and gradually moves to follow the present data for a low aspect ratio ($A = 20$) for the higher range of Ra. The departure from the pure conduction regime occurred exactly at the same Rayleigh number, and the turbulent flow regime occurred at $Ra \geqslant 8 \times 10^4$.

Figure 4 shows the data from the present study for $A = 20$ and 80 at $\phi = 150°$ and $\phi = 180°$. For $\phi = 150°$, again the flow changed from pure conduction at $Ra \approx 4000$ and the slope of the data increased until it reached the one third slope at $Ra \geqslant 2 \times 10^5$. This indicates that as ϕ was increased (getting closer to the horizontal layer heated from above), the change to the turbulent flow occurred at higher values of Ra. For $\phi = 180°$ (horizontal layer heated from above), the conduction regime prevailed up to $Ra = 10^4$. However, convection started to contribute to the heat transfer across the air layer as was expected by Catton *et al.* [3] due to the presence of perfectly conducting end walls. The dashed line representing the data from ref. [7] follows the present data very closely.

Additional experiments have been carried out to explain the effect of tilt angle on the average Nusselt number for fixed values of the Rayleigh number. The experimental results are plotted in Fig. 5 for $A = 20$ and the tilt angle was varied from 0 (heated from below) to 180° (heated from above). It is clear that the average Nusselt number decreased monotonically as the angle of tilt was increased from 0 to 180°. The scaling law derived by Arnold *et al.* [4] and given

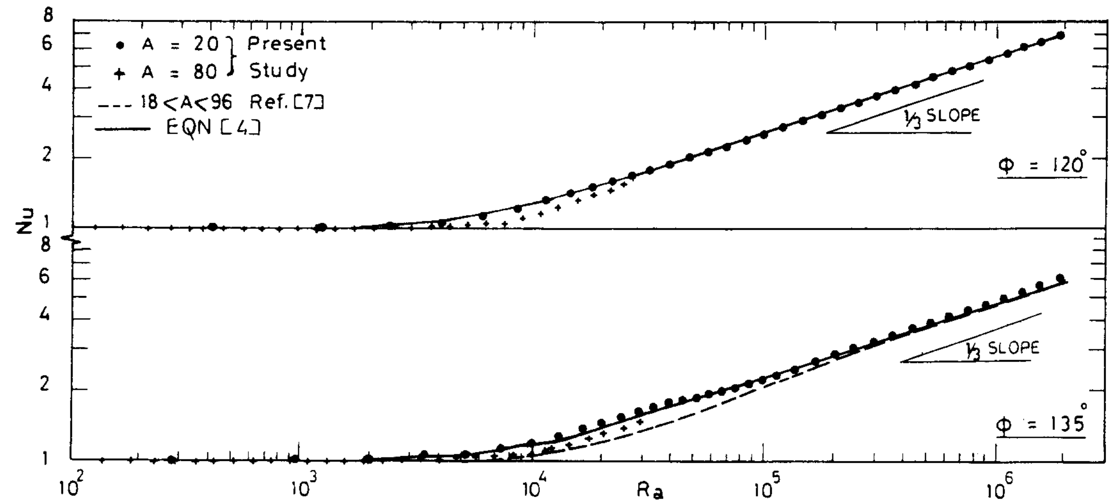

Fig. 3. Heat transfer measurements for ϕ = 120 and 135 degrees.

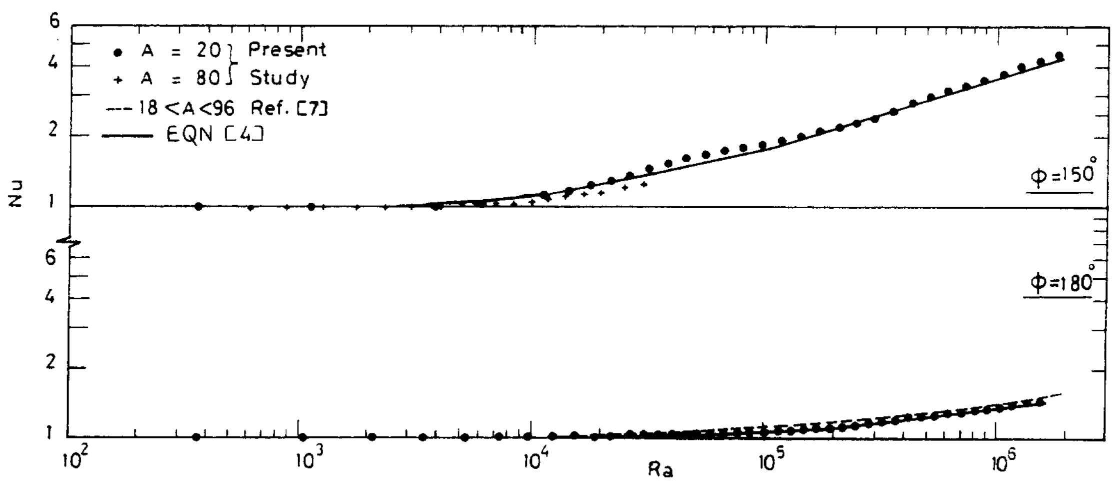

Fig. 4. Heat transfer measurements for ϕ = 150 and 180 degrees.

by equation (1) is drawn in Fig. 5 in its range of applicability ($90 \leqslant \phi \leqslant 180^{\circ}$). It is seen that the present data approximately follow the same scaling law, especially at high Rayleigh numbers, with a maximum deviation of less than 10%. Although the scaling law for $\phi = 180^{\circ}$ assumes totally pure conduction, the present data and those of ref. [7] show a real contribution of the convection due to the perfectly conducting boundary conditions on the end walls. For tilted layers heated from above, the average Nusselt number is considerably greater than that for pure conduction.

To the author's knowledge, no previous measurements were published for local Nusselt numbers in high aspect ratio air layers heated from above. The distribution of Nusselt number along the height of the air layer is seen in Fig. 6 for $A = 20$ and $\phi = 120^{\circ}$. The data represent the average Nusselt number on the lower, middle, and upper third of the hot plate. In the conduction regime, $Ra < 1000$, the three values were

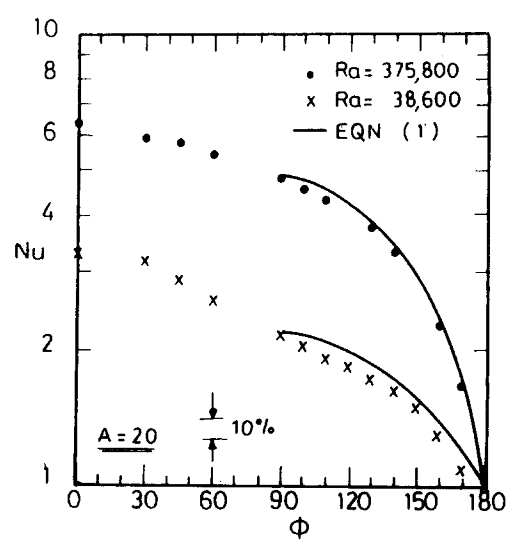

Fig. 5. Effect of tilt angle on Nu.

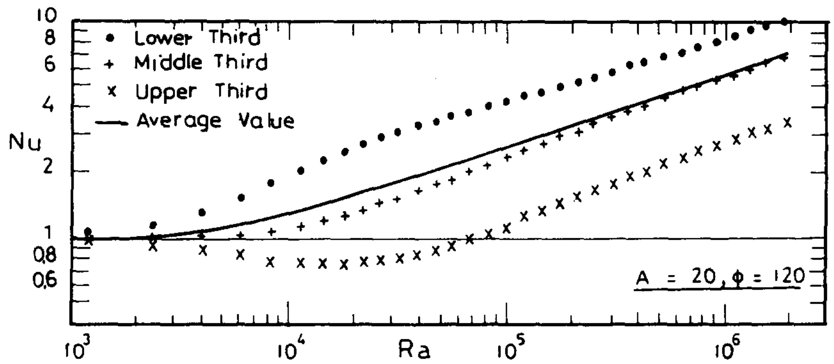

Fig. 6. Nusselt number distributions for $\phi = 120$ degrees.

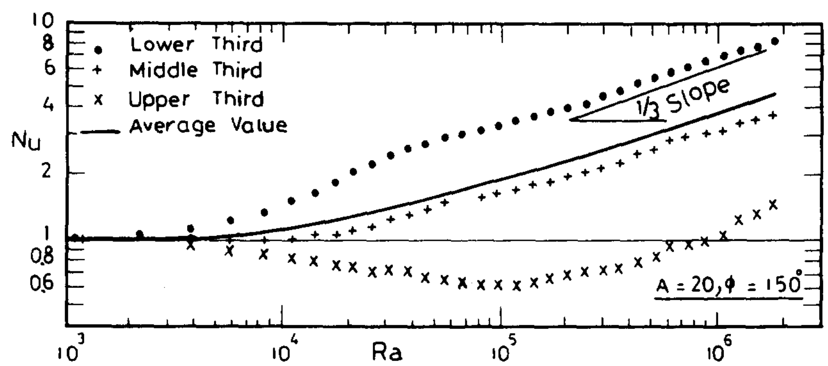

Fig. 7. Nusselt number distributions for $\phi = 150$ degrees

equal to unity. As Ra was increased, the average Nusselt number over the lower third of the hot plate increased very quickly and finally approached 1/3 slope at $Ra \geqslant 4\times10^5$. In the middle third of the layer, the average Nusselt number slowly increased and reached the same 1/3 slope at $Ra \geqslant 4\times10^5$. In the upper third of the hot plate, the average Nu continuously dropped below unity as Ra was increased until it reached a minimum value of 0.77 at $Ra = 1.45\times10^4$. A further increase in Ra caused the average Nu on the upper third to increase until it approached the same 1/3 slope at $Ra \geqslant 4\times10^5$. The average Nusselt number over the entire hot plate is plotted on the same figure as a solid line. It is clear that the average Nusselt number on the central portion of the hot-plate underestimates the average Nusselt number on the entire plate until Ra reaches a high value of 4×10^5. This indicates that for air layers heated from above, the measurement averaged over the central portion do not exactly represent measurements averaged over the entire plate unless the Rayleigh number is very high ($\geqslant 4\times10^5$).

Figure 7 shows the distribution of the Nusselt number along the hot plate for $A = 20$ and $\phi = 150°$. The same behaviour is repeated where the average Nu over the lower third quickly increased and then came back to the 1/3 slope at $Ra \approx 3\times10^4$. The data from the middle third of the hot plate fell below the average Nu for the entire plate. This continued for all values of Ra beyond the pure conduction regime. The average value of Nu for the upper third continuously dropped to a new lower minimum value of 0.62 at $Ra \approx 10^5$. Turning the air layer from $\phi = 120°$ to $\phi = 150°$ has the effect of damping the flow inside the layer for a fixed value of Rayleigh number. For $Ra > 10^5$ the average Nu for the upper third started to increase as Ra was increased.

4. DATA CORRELATIONS

The experimental measurements for $A = 20$ and $\phi = 180°$ were correlated using the method suggested by Churchill and Usagi [12] and the correlation is given as follows:

$$Nu = [1+(0.212Ra^{0.136})^{11}]^{1/11}. \qquad (2)$$

Equation (2) is graphically represented by a solid line in Fig. 4, where reproduction of data is excellent. The maximum deviation of Nu is 2.1% and the standard deviation is 1%.

The data for $A = 20$ and $\phi = 120°$ were correlated by the following equation:

$$Nu = [1+(0.0566Ra^{0.332})^{4.76}]^{1/4.76}. \qquad (3)$$

The maximum deviation of Nu is 2.9% and the standard deviation is 0.84%. Equation (3) is represented by a solid line in Fig. 3.

The effect of angle of tilt on the average Nusselt number for $A = 20$ and $120^\circ \leqslant \phi \leqslant 180^\circ$ can be expressed by the following linear relation:

$$Nu(\phi) = Nu(180^\circ) + \frac{180-\phi}{60}(Nu(120) - Nu(180)). \quad (4)$$

Equation (4) is shown in Figs. 3 and 4. It predicted the experimental data within a maximum deviation of $\pm 6.3\%$.

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